

*A Minimax Theory for Overdamped Networks**

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1. Introduction. Dynamical systems with friction are much more difficult to treat than conservative systems. It is the aim of this paper to show that a degree of simplicity is restored by considering the extreme case of large amounts of friction.

The theory of small vibrations of conservative systems is essentially the theory of a pair of definite quadratic forms: the kinetic-energy quadratic form a and the potential-energy quadratic form c . Two quadratic forms may be brought to diagonal forms simultaneously. This diagonalization defines the normal modes of vibration and their characteristic frequencies. It also reveals the orthogonality property of the normal modes.

The theory of small motions of systems with friction is the theory of the triplet of definite quadratic forms a , b , and c . Here b is the Rayleigh dissipation function. As RAYLEIGH [7] observed, the theory is complicated by the fact that it is usually impossible to diagonalize three quadratic forms simultaneously. It results that the normal modes of motion are not orthogonal.

In this paper dynamical systems are considered which are heavily damped. Such systems may be defined precisely by the inequality $b^2 - 4ac > 0$, termed the *overdamping* condition. This inequality is assumed to hold for any virtual mode of motion of the system. A normal mode for a conservative system is of the form qe^{ikt} where q is independent of the time t . A normal mode for an overdamped system is of the form qe^{kt} . In both cases k is real and may be termed the eigenvalue. (For intermediate amounts of friction, damped oscillations result. This would lead to complex eigenvalues. The overdamping condition rules out this type of behavior.)

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