

*First Order Partial Differential Equations Containing a Small Parameter**

J. J. LEVIN

Purdue University, Lafayette, Indiana

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1. Introduction. The equation investigated here is

$$(1.1) \quad \epsilon \left\{ a_1(x, y, u, \epsilon) \frac{\partial u}{\partial x} + a_2(x, y, u, \epsilon) \frac{\partial u}{\partial y} \right\} = b(x, y, u, \epsilon),$$

where ϵ is a small positive parameter and all the functions and variables appearing are real scalars. Conditions will be given under which the solutions of (1.1) passing through a fixed initial curve, Γ , where

$$(1.2) \quad \Gamma: \begin{cases} x = f_1(\tau) \\ y = f_2(\tau), \\ u = f_3(\tau) \end{cases} \quad \tau_0 \leq \tau \leq \tau_1;$$

approach the surface $b(x, y, u, 0) = 0$ as $\epsilon \rightarrow 0+$.

Our main result is Theorem 1, which is expressed in terms of (1.1) and is given below. In order to prove Theorem 1 it is convenient to transform (1.1) to a somewhat simpler form, (2.10). Theorem 1 is then restated, as Theorem 2, in terms of (2.10); this is done in Section 2. Some lemmas that are used repeatedly in the proofs are given in Section 3. In Section 4 the characteristic equations associated with (2.10) are investigated, and in Section 5, Theorem 2 is proven. A boundary layer equation associated with (2.10) is considered in Section 6.

The analysis centers around the characteristic curves associated with (2.10)

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