

# *The Explicit Solution of a Free Boundary Problem Involving Surface Tension*

E. B. MCLEOD, JR.

University of Colorado, Boulder, Colorado

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We shall treat the problem of finding the shape of a two-dimensional bubble having a significant surface tension. We assume the flow at infinity to be of uniform unit velocity without circulation. Since both the flow pattern and the cross-sectional shape of the boundary must be determined simultaneously, the problem is termed a *free boundary problem*. In order that the principles of conformal mapping be applicable we shall assume that the flow is irrotational and that the fluid is incompressible. The bubble shall be supposed to be filled with a gas that exerts an internal pressure,  $p_i$ . (To  $p_i$  we shall assign a specific value that will lead to a convenient mathematical simplification.) By employing Bernoulli's law we are led to a geometric condition that prevails on the surface of the bubble, or *free boundary*. From this geometric condition we deduce a conformal mapping which leads to a differential equation relating a point in the region exterior to the free boundary to a point exterior to the unit circle.

Like many free boundary problems this one has a variational analogy [3], [5]. It can be shown that the free boundary is a solution to the following variational problem:

Let the complex velocity potential,  $W$ , be represented by the expansion,

$$W = z + \frac{\alpha_1}{z} + \frac{\alpha_2}{z^2} \cdots,$$

and let  $a = \operatorname{Re}(\alpha_1)$ . Find the curve which has a minimum length, in a family of closed rectifiable competing curves each having a fixed value for the parameter,  $a$ .

The principal weapon for attacking such a problem is a method of comparison