

Approximately Planar Rigid Body Motion

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INTRODUCTION

Many cases of rigid body motion which are of practical interest consist in the spinning at high, substantially constant speed of a symmetrical body about an axis (angular velocity vector) which remains close both to the axis of symmetry of the body and to some fixed direction in space. Such a motion we shall call an approximately planar rigid body motion.

In this case, an approximate linearization of the equations of motion can be made. This linearization reduces the equations of motion to a single second-order linear differential equation and thus makes the integration trivial. The theory is sufficiently accurate to lead to a detailed prediction of many of the well-known phenomena of rigid body motion.

This paper first develops the equation and then applies it to some well-known classical cases.

I. GENERAL THEORY

A. Euler's equations. We begin by recalling Euler's equations for the motion of a rigid body having one point fixed. The derivation of Euler's equations is available in the standard texts on classical mechanics.

Let $(\mathbf{A}, \mathbf{B}, \mathbf{C})$, a moving orthonormal system of vectors, be the principal