

# *Explicit Remainder Terms for Some Asymptotic Series*

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*Communicated by C. TRUESDELL*

**Summary of results and notation.** AIREY and STIELTJES have indicated how to write asymptotic series (not always of the Poincaré type) for the remainder terms of the well known asymptotic series for the incomplete gamma function and related functions. As an illustration of some general techniques explained in the early part of our text, we give bounds on the errors for the series of AIREY and STIELTJES; in addition, we provide improved techniques for computing higher terms of the series.

We use  $t, u, v, w, x$  as variables to denote complex numbers, and  $j, k, l, m, n$  as variables (or parameters) to denote integers.  $M$  is the parameter of (1) or (24) in terms of which one seeks an asymptotic series. We preempt  $r, A, B, h(x)$ , and  $H(x)$  for their roles in (1) and (24);  $\phi$  for the unique constant determined by assumption (c) of Section 2;  $y, z$ , and  $Y$  as the parameters of the incomplete gamma function (first formula of Section 3) and of (48);  $N = m + 1 - z$ ;  $g_n(y)$  defined by (31) and (32);  $\gamma = y/N$ ;  $h_n(x)$  defined by (34) and (35);  $\delta = \gamma + \log \gamma$ ;  $\gamma(t)$  defined by (41);  $C(w)$  defined by (54);  $\alpha$  defined by (59);  $x(\alpha)$  defined by (63) so that  $x$  as a function of  $\alpha$  shall satisfy (59);  $A_j$  defined by (78);  $D_j$  defined by 82. There are also many transient notations that we have not listed, because they appear only in limited contexts.

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<sup>1</sup> The author wishes to make the following acknowledgements. Work on the present paper was begun while the author was at the Institute for Numerical Analysis, where his work was supported in part by the National Bureau of Standards and in part by the Office of Naval Research. Many of the numerical results of the paper were furnished by the computing staff of the Institute for Numerical Analysis. In particular the higher coefficients  $A_{2k+1}$  and the values in Table 1 were computed by SHIRLEY MARKS under the direction of Dr. GERTRUDE BLANCH. Work on the present paper was concluded while the author held a Fulbright Research Grant and a Guggenheim Fellowship, during which time the facilities of the Institut Henri Poincaré of the University of Paris were made available to him.