

The Differential Equations of the Stream Lines for Compressible Gas Flow

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1. Introduction. In the following paper¹ we have derived the differential equations giving the direct determination of the stream line pattern for the case of the steady flow of a compressible gas in a plane under the assumption that the viscosity and thermal conductivity are zero. This is accomplished by the successive elimination of the pressure and the density by which we obtain a system of two equations in the velocity alone. Following ERICKSEN² the velocity components are now replaced by the inclination ω of the stream lines and the quantity $G = m^2 - 1$ where m is the local mach number. At each stage of this process it is observed that the equations obtained are fully equivalent to the original system governing the flow of the gas.

The condition of integrability of the above system is a cubic equation in the quantity G with coefficients which are scalar differential functions of the inclination ω . Hence G can be expressed as an algebraic function of ω and its derivatives. By elimination of G we arrive at a system of two non-linear differential equations in the single dependent variable ω . However, these equations, as shown in §10, are not fully independent. Either equation can be taken as the equation to determine the stream line pattern after which the other equation will effectively impose only an additional boundary condition.

The solution of the Cauchy problem for the stream line pattern is given in §12. Characteristic curves are discussed and determined in §13 and these turn out, as might be expected, to be the stream lines and those curves making an angle μ , where $\sin \mu = 1/m$, with the stream lines. Finally we have given the formal solution of the problem of determining the stream lines when the data

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² J. L. ERICKSEN, *On the uniqueness of ideal gas flows with given stream line patterns*, Bulletin of the Technical University of Istanbul, vol. 6, 1953, pp. 1-5.