

Contour Integral Solutions of a Class of Differential Equations

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1. Introduction. This paper is concerned with representations as contour integrals of certain exact solutions of a class of partial differential equations and other forms of solutions to which these lead. One advantage of such representations, both as real and contour integrals, is that they permit the writing down of a general solution containing arbitrary functions which occur within the integral. Much of this advantage is lost, however, if the task of relating these arbitrary functions to prescribed boundary conditions of physical and mathematical interest is prohibitively difficult. Many well-known examples could be quoted of real integrals where this problem is resolved by elegant applications of integral transform techniques. The inversion from a contour integral is of less frequent occurrence but in the work which follows it will be shown how boundary value problems of significance in their physical applications can be handled with success, this success arising largely from the solution of a certain contour integral equation which is described at the beginning of §2. A point of mathematical interest is the exploitation of known solutions of a hyperbolic equation to provide solutions of an elliptic equation, related to it by a complex transformation of the independent variables. In fact this whole approach is largely motivated by a method which COPSON used [2] to solve a specific boundary value problem in the theory of one-dimensional, unsteady, isentropic flow of a compressible gas. A further application of this technique was later used by the present author [5]. The equation to be solved in these cases was the well-known gas dynamics equation of Euler-Poisson type

$$\frac{\partial^2 w}{\partial r \partial s} + \frac{N \left(\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} \right)}{r + s} = 0. \quad (1.1)$$