

*On the Convergence of an Approximation  
Method of M. J. Lighthill\**

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**Introduction.** In [1] M. J. LIGHTHILL proposed a new perturbation technique for the approximate solution of certain types of non-linear differential equations near their singular points. This method has since found numerous useful applications in hydrodynamical problems, an excellent account of which can be found in a report by H. S. TSIEN [2]. The procedures are justified, in the existing literature, by very convincing plausibility considerations, but mathematically complete proofs seem to be lacking.

The purpose of this note is to give such a proof in the simple but typical case of the differential equation

$$(x + \alpha u) \frac{du}{dx} + q(x)u - r(x) = 0, \quad (1.1)$$

where  $\alpha$  is a small positive parameter. This is the example used also in [1] for an introduction to the technique. In fact, our proof is essentially a completion of the order-of-magnitude arguments employed in that article. We shall therefore be somewhat brief here in our explanation of the formal aspects of the procedure.

The problem is to represent the solution of (1.1) through a point

$$x = a, \quad u = b \quad (1.2)$$

in a form that remains valid near  $x = 0$ ,  $\alpha = 0$ . This is achieved by referring  $x$  and  $u$  to a suitably chosen parameter  $\xi$ , as will be explained in the next section.

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