

On Asymptotically Linear Second-order Oscillations

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1. Statement of the problem. In this paper I discuss the problem of finding asymptotic formulae as $t \rightarrow \infty$ for bounded solutions of the second-order differential equation, in the real domain,

$$\ddot{x} + xF(x, t) = 0, \quad (\dot{} = d/dt), \quad (1.1)$$

which is to be asymptotically linear and oscillatory in the sense that $F(x, t) \rightarrow 1$ as $t \rightarrow \infty$ for any fixed x . Making certain assumptions about the smoothness of $F(x, t)$, bounded solutions will certainly exist, and the problem reduces to that of determining a function ω such that a bounded solution of (1.1) admits as $t \rightarrow \infty$ the asymptotic representation

$$x = A \cos \left(\int_{t_0}^t \omega dt + B \right) + o(1), \quad (1.2)$$

where A and B are constants for the solution in question. The oscillations being non-linear, it will not in general be possible to have ω the same for all bounded solutions; it turns out however that the same ω will serve for solutions with the same asymptotic amplitude A , and the problem will be to determine ω as a function of t and A . I show here that this problem is soluble with certain assumptions about the smoothness of $F(x, t)$, and without any requirements as to the rapidity with which $F(x, t) \rightarrow 1$ as $t \rightarrow \infty$.

This problem may be viewed as a particular case of the general theory of the comparison of the n -dimensional systems

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y}, \quad (1.3)$$

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{f}(\mathbf{y}, t), \quad (1.4)$$