

On a Class of Partial Differential Equations

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1. INTRODUCTION

1.1. The problem considered in the present paper. One of the efficient tools in the study of the harmonic equation $\Delta\psi = 0$ is the theorem that the harmonic polynomials

$$\operatorname{Re}(x + iy)^n, \quad \operatorname{Im}(x + iy)^n, \quad n = 0, 1, \dots,$$

represent an infinite set of independent particular solutions of this equation. It is natural in passing to other partial differential equations in two variables to determine an infinite set of independent particular solutions which can be characterized in a simple manner, *e.g.*, by the fact that each of these solutions satisfies an ordinary differential equation, and to draw from this fact various conclusions.

In the present paper we shall consider a class of partial differential equations possessing particular solutions which satisfy not only the partial differential equation itself but also ordinary differential equations with algebraic coefficients. The existence and the form of these ordinary differential equations will be investigated in a later paper. In this way we can use the theory of ordinary differential equations as an efficient tool when investigating the properties of partial differential equations. The construction of the desired particular solutions of the partial differential equations will be done by the aid of BERGMAN's operators (see definition below).

Obviously if instead of a linear space of functions one considers an algebra (*i.e.*, a set of functions such that addition and multiplication reproduces the functions belonging to the set) one obtains various new relations and one can gain a deeper insight into the laws governing the class under consideration. The basic idea of the BERGMAN theory of operators consists in establishing different