

*On the Exact Relation Between the  
Specific Heats of an Elastic Solid*

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*Communicated by C. TRUESDELL*

**1. Introduction.** The relation between the specific heats at constant volume and at constant pressure is usually claimed to hold not only for fluids but also for solids<sup>1</sup>. The meaning of this generalisation, however, is not very clear, and its proof cannot be considered satisfactory, when, as is the case, nothing is said about the state of the solid body which is chosen as a reference or about the value of the pressure at which the specific heat is computed. In fact, the exact character of such a generalisation and its validity have even been questioned<sup>2</sup>, and there are some who consider it nothing more than a good approximation, valid when the thermal expansion is constant and the specific heat is independent of volume or pressure.

Obviously the correct relation between the specific heats of solids can be obtained only from the general theory of finite thermoelastic deformations. As is shown in the present paper, the relation can be derived without introducing any hypothesis on the type of strain energy, by considering only a certain invariance requirement which characterizes the dependence of the thermoelastic potential upon temperature. A new proof of this requirement, making very evident its physical meaning, is given; the difference of the specific heats is then computed, and it is found that the well known relation for fluids holds exactly also for homogeneous isotropic elastic bodies, when they are in a natural equilibrium state, entirely free from stress.

**2. An invariance requirement for the thermoelastic potential.** The thermoelastic potential (or free energy)  $F$  of an elastic solid in its actual state  $C$ , at the uni-

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<sup>1</sup> See for example: M. PLANCK: *Treatise on Thermodynamics*, Dover, 1945, pp. 59, 126; E. A. GUGGENHEIM: *Thermodynamics*, North Holland Publishing Co., 1950, p. 87.

<sup>2</sup> See J. C. SLATER: *Introduction to Chemical Physics*, McGraw Hill, 1939, p. 205.