

Solution of Some Problems in the Exact Theory of Visco-Elasticity

R. S. RIVLIN

1. Introduction. It has been shown in a previous paper (RIVLIN, 1955), that if we assume that in a visco-elastic fluid, which is isotropic in its state of rest, the stress components t_{ij} in a rectangular Cartesian coordinate system x_i at a point x_i of the fluid are expressible as polynomials in the gradients of velocity, acceleration, second acceleration, \dots , $(n - 1)^{\text{th}}$ acceleration at the point considered, then the stress matrix \mathbf{T} ($= || t_{ij} ||$) may be expressed as a polynomial in n kinematic matrices $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n$. The coefficients in this polynomial are scalar invariants of the matrices $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n$ and are expressible as polynomials in traces of products formed from these matrices. The kinematic matrices $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n$ are defined by the equations

$$\mathbf{A}_r = || A_{ij}^{(r)} || \quad (r = 1, 2, \dots, n), \quad (1.1)$$

$$A_{ij}^{(1)} = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}, \quad (1.2)$$

$$A_{ij}^{(r+1)} = \frac{\partial A_{ij}^{(r)}}{\partial t} + v_l \frac{\partial A_{ij}^{(r)}}{\partial x_l} + A_{mi}^{(r)} \frac{\partial v_m}{\partial x_j} + A_{mj}^{(r)} \frac{\partial v_m}{\partial x_i} \quad (1.3)$$

$$(r = 1, 2, \dots, n - 1),$$

where v_i is the velocity of the fluid at the point x_i .

In the present paper we shall consider the forces necessary to support in such fluids certain simple types of steady-state laminar flow. These are:

- (i) rectilinear laminar flow;
- (ii) torsional flow between two parallel discs;
- (iii) helical flow in the annular region between two coaxial cylinders.

It will be assumed throughout the paper that the fluid is incompressible and it may be remarked that the types of flow considered do not violate this con-