

*On an Initial Value Problem for Flow of a
Viscous Incompressible Fluid
in an unbounded Region*

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1. Introduction. The motion of an incompressible, viscous fluid is governed by the Navier-Stokes equation and the continuity equation. As is well known, the analysis of such fluid motion is greatly complicated by the fact that the Navier-Stokes equation is non-linear. It has been recognized that when a solution to the flow equations is sought in terms of a Fourier representation for the velocity field, the various Fourier components will interact with each other as a consequence of the non-linearity of the equation of motion. It is the purpose of this article to study this interaction and to apply the results of the investigation to an initial value problem.

For simplicity, we consider only an unbounded flow. We shall be concerned with a special case of the following problem: To investigate the subsequent development of the motion of an unbounded viscous incompressible fluid, when initially its motion is described by two spatially periodic waves. The solution of this problem will be carried out by successive approximation, and the existence of the solution will be proved. We shall first express the velocity and pressure fields in a Fourier representation and then transform the Navier-Stokes and continuity equations into equations for the Fourier components.

2. Transformation of the Navier-Stokes and Continuity Equations. To express the velocity and pressure field in a convenient trigonometrical representation, we introduce the following function:

$$K(\mathbf{r} \cdot \mathbf{x}) = \cos(\mathbf{r} \cdot \mathbf{x}) + \sin(\mathbf{r} \cdot \mathbf{x}),^* \quad (2.1)$$

where \mathbf{r} is a wave number vector and \mathbf{x} is a position vector in the physical space. By permitting \mathbf{r} to be negative in (2.1), we see that a trigonometrical expansion

* \mathbf{x} denotes a vector and x its magnitude.