

# *On the Use of the Kirchhoff Approximation for the Solution of Reflection Problems*

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1. **Introduction.** In the past there has been a considerable amount of work done on the problem of the reflection of radiation from non-plane (frequently termed irregular) surfaces. Of the various approximations used in approaching this problem, perhaps the most popular is the one due to KIRCHHOFF [1]. (For work using this approximation see references [2], [3], and [6].) It is assumed in this approximation that the field near every region of the surface is essentially what it would have been if the surface had been flat with a slope equal to that of the irregular surface at the point in question. The approximation has been assumed to be useful when the surface variations are in some sense large compared with the wavelength of the incident radiation. Using this assumption concerning the value of the field at the bounding surface in conjunction with the Helmholtz formula (see below) it is possible to obtain an estimate of the field in regions removed from the reflecting surface. Despite the prevalence of the Kirchhoff approximation in the treatment of the problem of the reflection of radiation from an irregular surface there does not exist, to the author's knowledge, a systematic derivation of the approximation which demonstrates the way in which it may be corrected, nor for that matter, which shows conclusively the size of the errors incurred through its use. For this reason it is hoped that the derivation in the present paper may prove of some interest.

For simplicity consideration will be restricted to two dimensional problems in which the field function is assumed to vanish at the irregular surface (in acoustical work, a pressure release surface). It is a simple matter to extend the arguments to cover the case where the normal derivative of the field vanishes at the surface as well as to extend them to cover analogous three dimensional (scalar) problems. One wishes to find then a function  $\Phi(x, z, t)$  which satisfies the wave equation in two dimensions,

$$(1.1) \quad \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2},$$