

An Isoperimetric Inequality for the N-Dimensional Free Membrane Problem

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1. This paper is concerned with the eigenvalue problem

$$(1.1) \quad \Delta u + \mu u = 0 \quad \text{in } R,$$

$$(1.2) \quad \partial u / \partial n = 0 \quad \text{on } \dot{R}.$$

R is a region in N -dimensional Euclidean space and \dot{R} is its boundary. If the eigenvalues are arranged in non-decreasing order

$$(1.3) \quad \mu_1 \leq \mu_2 \leq \cdots ,$$

it is easily seen that

$$(1.4) \quad \mu_1 = 0.$$

The corresponding eigenfunction is $u = \text{constant}$. Thus, μ_2 is the first eigenvalue of interest.

For $N = 2$, μ_2 is proportional to the square of the lowest resonant frequency of vibration of a membrane R whose edges are free. It is also proportional to the square of the cutoff frequency of the lowest H -mode of a wave guide of cross section R [1]. For $N = 3$, μ_2 is proportional to the square of the lowest resonant frequency of an acoustical resonator R with perfectly rigid walls.

The following assertion was made by E. T. KORNHAUSER & I. STAKGOLD [1] and proved by G. SZEGÖ [2]: *For all two dimensional simply connected domains R of given area, the circle yields the maximum value of μ_2 .* The proof of SZEGÖ leans heavily upon conformal mapping and hence cannot be extended to more than two dimensions.

In this paper we prove the following generalization of the above assertion: