

*On the Completeness of the Boussinesq-Papkovich Stress Functions**

R. A. EUBANKS & E. STERNBERG

1. Introduction.

In the absence of body forces, the displacement equations of equilibrium in the linear theory of homogeneous and isotropic elastic media, appear as

$$\nabla^2 \mathbf{u} + \frac{1}{1-2\nu} \nabla \nabla \cdot \mathbf{u} = 0, \quad (1.1)^1$$

where \mathbf{u} is the displacement field, while ν denotes Poisson's ratio.

PAPKOVICH [1],² [2] in 1932 established the general solution of (1.1) in the form

$$\mathbf{u} = \nabla(\varphi + \mathbf{r} \cdot \boldsymbol{\psi}) - 4(1-\nu)\boldsymbol{\psi}, \quad (1.2)$$

where

$$\nabla^2 \varphi = 0, \quad \nabla^2 \boldsymbol{\psi} = 0. \quad (1.3)$$

Here \mathbf{r} is the position vector of a field point, referred to the origin O . Substitution into (1.1) at once confirms that the vector field (1.2) satisfies the equilibrium equations provided that the generating stress functions (displacement potentials) φ , $\boldsymbol{\psi}$ are sufficiently smooth and meet (1.3). Conversely, every sufficiently regular solution of (1.1) admits the representation (1.2), (1.3), as shown by PAPKOVICH (*loc. cit.*). A more direct proof of the completeness of this representation is due to MINDLIN [3].³

*The results communicated in this paper were obtained in the course of an investigation conducted under Contract N7onr-32906 with the Office of Naval Research, Department of the Navy, Washington, D. C.

¹Throughout this paper bold face letters designate vectors; the symbols “ \cdot ” and “ \times ” indicate scalar and vector multiplication of two vectors, respectively. ∇ is the usual del-operator.

²Numbers in brackets refer to the bibliography at the end of the paper.

³See also SOKOLNIKOFF [4], p. 329 *et seq.*, where the same argument is presented. MINDLIN [5] recently extended his proof to the case where the body forces are included; in this instance the Laplace equations (1.3) need to be replaced by appropriate Poisson equations.