

# *On Certain Partial Differential Equations and Their Singularities*

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**1. Introduction.** We investigate the partial differential equation

$$(1.1) \quad \Delta\psi + \alpha(x, y) \frac{\partial\psi}{\partial x} + \beta(x, y) \frac{\partial\psi}{\partial y} + \gamma(x, y)\psi = 0,$$

whose coefficients  $\alpha$ ,  $\beta$ , and  $\gamma$  are analytic functions of the real variables  $x$  and  $y$ . As is known, for every linear partial differential equation it is possible to generate various systems of particular solutions; see *e.g.* BERGMAN [1]. It is of interest to obtain systems of independent particular solutions having special characteristic properties, for instance, solutions which satisfy also an ordinary differential equation with rational or algebraic coefficients. In the present paper we shall determine such systems of solutions for a certain class of partial differential equations (1.1). In order to do so we first continue the functions  $\alpha$ ,  $\beta$ , and  $\gamma$  to complex values of the variables. We write  $z_1 = x + iX$ ,  $z_2 = y + iY$  and then introduce the variables  $z = z_1 + iz_2$ ,  $z^* = z_1 - iz_2$ . For complex values of  $z_1$  and  $z_2$  the variables  $z$  and  $z^*$  are independent of each other while for real values, *i.e.* for  $z_1 = x$ ,  $z_2 = y$ , the variable  $z^*$  is conjugate complex to  $z$ , *i.e.*,  $z^* = \bar{z}$ . Using the notations

$$\frac{\partial}{\partial z} = \frac{1}{2} \left( \frac{\partial}{\partial z_1} - i \frac{\partial}{\partial z_2} \right), \quad \frac{\partial}{\partial z^*} = \frac{1}{2} \left( \frac{\partial}{\partial z_1} + i \frac{\partial}{\partial z_2} \right),$$

(see *e.g.* BERGMAN [1]) we write (1.1) in the form

$$(1.2) \quad w_{zz^*} + a(z, z^*)w_z + b(z, z^*)w_{z^*} + c(z, z^*)w = 0, \quad w_{zz^*} \equiv \frac{\partial^2 w}{\partial z \partial z^*} \quad \text{etc.},$$