

On Solutions of Riccati's Equation as Functions of the Initial Values

RAYMOND REDHEFFER

Communicated by A. ERDÉLYI

Introduction. Subject to mild restrictions on a , b , and c , the general Riccati equation

$$\frac{du}{dy} = a(y) + 2b(y)u(y) + c(y)u^2(y)$$

has a solution through any given point of the (u, y) plane. In particular, one may consider that solution which satisfies the initial condition

$$u = 0 \quad \text{when} \quad y = x$$

or indeed

$$u = f(x) \quad \text{when} \quad y = x$$

where $f(x)$ is given.

The function u so obtained depends on x , naturally, as well as on y , and our notation emphasizes this feature. In view of the x -dependence it is appropriate to write

$$u = u(x, y)$$

rather than simply $u = u(y)$. We have, then,

$$u_y \equiv \frac{\partial u}{\partial y} = a(y) + 2b(y)u(x, y) + c(y)u^2(x, y)$$
$$u(x, x) = f(x).$$

This paper was prepared under the sponsorship of the Office of Naval Research and the Office of Ordnance Research. Reproduction in whole or in part is permitted for any purpose of the United States Government.