

# *Analytic Continuation in Elasticity*

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## 1. Introduction.

Of concern here are the equations of static equilibrium for an isotropic elastic body. This system of partial differential equations of the second order governs the three components of the displacement vector. In the first part of this paper formulæ are developed which express the displacements and the stresses in terms of three harmonic functions. (The proof is restricted to certain cylindrical regions.) These formulæ are suited for the study of reflection principles in elasticity.

The reflection principle of SCHWARZ applies to a function which is harmonic in a region and which vanishes on a plane boundary surface of the region. The function can be continued analytically across the surface by considering the reflection of a point in this plane and ascribing the same absolute value of the function at the reflected point but the opposite sign. This principle together with the representation formulæ mentioned above are combined to yield analytic continuation of solutions of the equations of elasticity. It is assumed, of course, that one of the boundary surfaces of the elastic body is planar. Continuation formulæ are developed for the case of a fixed planar boundary and for the case of a free planar boundary. A boundary is termed "fixed" if the displacement vanishes there and a boundary is termed "free" if the surface tractions vanish there. Boundary conditions involving a mixture of such restrictions are also treated.

This paper may be regarded as a sequel to a previous paper concerning analytic continuation of biharmonic functions [2]. The techniques employed here are patterned after those developed in that paper. In addition to the references cited in the previous paper the works of DIAZ & LUDFORD [1], HUBER [6], and JOHN [7] are here noted. As in the previous paper the boundary conditions are expressed as suitable limiting relations as the boundary is approached from the interior. This procedure avoids *a priori* hypotheses on the existence of derivatives at boundary points.