

On the Propagation of Weak Discontinuities in Perfectly Plastic Solids

T. Y. THOMAS

1. Introduction. The concept of the singular surface which was used extensively by Hadamard¹ at the beginning of this century in his researches on the propagation of waves has recently been applied by J. L. ERICKSEN and the present writer² to derive conditions for the existence of singular or characteristic surfaces in the von Mises and the Prandtl-Reuss theories of plasticity.³ These surfaces are associated with a *weak* type of discontinuity of the solution functions of the problem in that only discontinuities in the derivatives of the functions occur, the functions themselves being continuous across the surfaces in question. Surfaces over which the actual functions are discontinuous represent *shocks* or *strong* discontinuities.

In the following paper⁴ we have investigated the existence of *singular surfaces of order 1*, also referred to as *wave surfaces*, in the plane stress problem for perfectly plastic solids, under the assumption of the von Mises and the Prandtl-Reuss equations of plastic flow. The interesting feature of this work is that the additional conditions due to the requirement of plane stress enable us, in general, to arrive at an *algebraic* determination of the direction of propagation of the singular or wave surface, *i.e.*, its unit normal vector, as well as the actual speed of propa-

¹ See, J. HADAMARD, *Leçons sur la Propagation des Ondes et les Équations de l'Hydrodynamique*, Paris, 1903.

² J. L. ERICKSEN, *Singular surfaces in plasticity*, J. Math. and Physics **34** (1955) pp. 74–79. Also, T. Y. THOMAS, *On the characteristic surfaces of the von Mises plasticity equations*, J. Rat. Mech. and Anal. **1** (1952) pp. 343–357; *Singular surfaces and flow lines in the theory of plasticity*, *ibid.* **2** (1953) pp. 339–381; and *Characteristic surfaces in the Prandtl-Reuss plasticity theory*, *ibid.* **5** (1956) pp. 251–262. A discussion of discontinuities in plastic flow with applications and numerous references has been given by W. PRAGER, *Discontinuous fields of plastic stress and flow*, Proc., 2nd U.S. National Congress of Appl. Mech., Ann Arbor, 1954, pp. 21–32; see also, *The theory of plasticity: A survey of recent achievements*, Proc. Inst. Mech. Engrs. **169** (1955) pp. 41–57.

³ For a general discussion and application of the von Mises and Prandtl-Reuss theories of plasticity see, W. PRAGER and P. G. HODGE, JR., *Theory of Perfectly Plastic Solids*, J. Wiley and Sons, New York, 1951.

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