

Extended Asymptotic Eigenvalue Distributions for Bounded Domains in n -Space

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1. Introduction

Let D be a bounded, open, connected set with boundary B in R_n , n -dimensional Euclidean space. Letting $\nabla^2 u$ denote the Laplacian and ∇u the gradient of u , consider the problem with eigenvalue λ

$$(1.1) \quad \nabla^2 u + \lambda u = 0 \quad \text{on } D,$$

where $u(\mathbf{x})$ is to be real-valued with continuous second partials over $\mathbf{x} \in D$. For the Dirichlet condition u is to possess a continuous extension over the closure \bar{D} such that

$$(1.2) \quad u = 0 \quad \text{on } B.$$

For the Neumann condition ∇u is to possess a continuous extension over \bar{D} such that its normal component

$$(1.3) \quad \frac{\partial u}{\partial n} = 0 \quad \text{on } B,$$

at least if B is sufficiently smooth; if not, (1.3) will be appropriately modified (see (2.3)).

Let $\nu\lambda_i \leq \nu\lambda_{i+1}$ be the eigenvalues of the first of these problems if $\nu = 0$ and of the second if $\nu = 1$, the $\nu\lambda_i$ being repeated according to their multiplicities. We want to study the asymptotic behavior of $N_\nu(\lambda) = \sum_{\nu\lambda_i \leq \lambda} 1$, the number of eigenvalues $\leq \lambda$, as $\lambda \rightarrow +\infty$. The original result of WEYL was later obtained by CARLEMAN for dimension $n = 2$ or 3 by applying Tauberian theorems to an asymptotic evaluation of $\sum_{i=1}^{\infty} \nu\lambda_i^{-1} (\nu\lambda_i + \omega^2)^{-1}$ as $\omega \rightarrow +\infty$. This evaluation was found by utilizing the Green function of the "meson" equation operator $-\nabla^2 + \omega^2$, ([16], see bibliography). In two recent papers [13] and [14] PLEIJEL has found, in addition to CARLEMAN's leading term, the asymptotic series for $\sum_{i=1}^{\infty} \nu\lambda_i^{-1} (\nu\lambda_i + \omega^2)^{-1}$ in powers of ω^{-1} to any desired order for $n = 2$ or 3 . This work assumes the boundary B to be infinitely differentiable everywhere,