

The Existence and Uniqueness of the Velocity Correlation Derivative in Chandrasekhar's Theory of Turbulence

GEORGE BACKUS

Communicated by T. Y. THOMAS

CONTENTS

1. Introduction	215
2. The general plan of attack	216
3. Preliminary mathematical remarks	218
4. The singularity at $x = 0$	219
5. Singular points on $9 + 4x^2\phi = 0$	223
6. The singular point at $x = \infty$	226
7. An examination of equation (7) in the large	229
8. Conclusion	231
9. Appendix	232
References	233

1. Introduction. For an isotropically turbulent, incompressible, random fluid-velocity field CHANDRASEKHAR [1] has closed at the fourth moments the infinite sequence of moment equations analogous to the Karman-Howarth equation by assuming that certain correlations of four velocities are bilinear forms in the correlations of pairs of velocities, as if the four relevant velocities had a joint normal distribution. He obtains a fifth-order, non-linear partial differential equation in the velocity correlation $f(r, t) = \langle u_1(\mathbf{x} + \mathbf{r}, \tau + t) u_1(\mathbf{x}, \tau) \rangle_{av}$ (CHANDRASEKHAR [2]). The attempt to specify boundary conditions on f by applying to it Kolmogoroff's similarity principles leads to such difficulties that CHANDRASEKHAR concludes that these similarity principles apply not to f but only to $\chi(r, t) = -\partial f / \partial r$. In the limit of zero viscosity Kolmogoroff's prin-

The research reported in this paper has in part been supported by the Geophysics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command, under Contract AF 19(604)-299 with the University of Chicago.