

Extended Compatibility Conditions for the Study of Surfaces of Discontinuity in Continuum Mechanics

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1. Introduction. Suppose that the dependent variables of a dynamical problem, *e.g.* the pressure p , the density ρ , *etc.*, are continuous across a moving surface $\Sigma(t)$ but that discontinuities in the coordinate and time derivatives of these variables can occur over the surface. If we denote the discontinuities in the derivatives by means of a bracket $[\]$ as is customary, then, using the pressure for illustration, it is well known that we must have relations of the form

$$(1) \quad [p, i] = \xi \nu_i ; \quad [\partial p / \partial t] = -\xi G$$

over $\Sigma(t)$, where G denotes the velocity of this surface in the direction of the unit normal ν and ξ is some function defined over the surface. When the problem involves a system of differential equations of the first order, the above relations, together with the system of differential equations, yield the condition for $\Sigma(t)$ to be a characteristic surface provided the function ξ associated with the pressure p in the relations (1), or a corresponding function connected with one of the other dependent variables, does not vanish so that discontinuities in the derivatives necessarily occur over $\Sigma(t)$. Actually the use of relations of the general type (1) in the determination of characteristic surfaces for systems of differential equations of the first or higher order avoids some of the details of calculation which arise in the common Cauchy procedure.

Conditions analogous to (1) have been derived by HADAMARD¹ on the basis of the Lagrangian parameters rather than the space coordinates as Eulerian variables which we prefer. He has also considered an extension of such relations giving the discontinuities in the second and higher derivatives of quantities under the assumption that only the quantities themselves are continuous over $\Sigma(t)$. However these latter relations, like the former, involve the Lagrangian parameters and are, moreover, in a form which does not appear suitable for application.

¹J. HADAMARD, *Leçons sur la Propagation des Ondes et les Équations de l'Hydrodynamique*, Paris, 1903.