

# *Relations Between Properties of Solutions of Partial Differential Equations and the Coefficients of their Power Series Development*

ERWIN KREYSZIG

*Communicated by S. BERGMAN*

## 1. INTRODUCTION

**1.1. Problems considered in this paper.** The theory of solutions of linear partial differential equations of elliptic type and with analytic coefficients,

$$(1.1) \quad \Delta v + \alpha(x, y)v_x + \beta(x, y)v_y + \gamma(x, y)v = 0,$$

can be based upon the theory of functions of a complex variable, as S. BERGMAN has recently shown in a number of important papers, *cf.* [1–6] of the bibliography at the end of this paper. Obviously this relation is a generalization of the well known correspondence between analytic and harmonic functions.

The present paper is concerned with two closely related problems. In Section 2 we shall obtain relations between properties of solutions of (1.1) and those of the coefficients of the corresponding power series development. By means of appropriate Bergman operators (definition below) theorems of that kind can be derived from well known similar theorems on analytic functions  $f(z)$ ,  $z = x + iy$ . We restrict ourselves to some interesting problems which may arise in this connection. We first consider singularities of solutions of (1.1) and representations of these solutions in terms of particular solutions which are “most simple” in a sense defined in Section 2.1. Section 2.2 is devoted to solutions of (1.1) which satisfy also ordinary differential equations. This property is very advantageous since it permits the investigation of these solutions by means of the theory of ordinary differential equations. We shall see that the results thus obtained depend only on a certain subset of the coefficients in the power series development of the solutions. This fact leads to the important question as to what extent information on the behavior of a solution can be obtained from other subsets of those coefficients. In Section 3 we shall find relations between such subsets; if the coefficients of (1.1) are entire functions these relations either are completely independent of  $\alpha$ ,  $\beta$ , and  $\gamma$  or depend on these func-