

Stability Theorems for a Class of Nonlinear Servomechanisms

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1. Statement of the problem. Let us consider real differential systems of the form

$$(1) \quad \dot{x}^i + f^i(x) = h^i(z, \dot{z}) \quad (i = 1, 2, \dots, n)$$

where f and h are smooth functions and the vector $z(t)$ is selected from some known class I of input functions. We seek conditions on f , h , and I so that the solutions or output vectors $x(t)$ shall have certain stability properties.

A simple example is the servomechanism constructed by suspending a mass m between two springs on a horizontal line. The first spring has one end fixed to the line, say at the origin, and the coordinate of the mass m at time t is $x(t)$. The second spring has one end fastened to a moving point with coordinate $z(t)$. If the first spring has a nonlinear restoring force $-(kx + \beta x^3)$, whereas the second spring produces a linear force $a(z - x)$, and if there is a frictional force $-c\dot{x}$ on m , then the differential equation for the motion of m is

$$m\ddot{x} + c\dot{x} + kx + \beta x^3 = a(z - x),$$

with m , c , k , β , a positive constants. The stability theory for such equations will always refer to that of the equivalent first order system. Here let $x^1 = x$, $x^2 = \dot{x}$, $z^1 = z$ and write

$$\begin{aligned} \dot{x}^1 - x^2 &= 0 \\ \dot{x}^2 + \frac{c}{m} x^2 + \frac{k+a}{m} x^1 + \frac{\beta}{m} (x^1)^3 &= \frac{a}{m} z^1. \end{aligned}$$

If $z = s$ is a constant, then it is physically clear that each solution $(x(t), \dot{x}(t)) = x^i(t)$ of this system approaches a certain equilibrium value as $t \rightarrow +\infty$. If $z^i(t)$ is not constant but varies quite slowly, one expects the output $x^i(t)$ to "follow" the input signal $z^i(t)$. That is, fluctuations in $z^i(t)$ will be reproduced, after some specified modification, as fluctuations of $x^i(t)$. We shall formalize this problem and prove a general theorem which yields the desired result for the spring example as a special case.