The Growth and Decay of Sonic Discontinuities in Ideal Gases

T. Y. THOMAS

1. Introduction. We shall be concerned in this paper[1] with an ideal gas, devoid of viscosity and thermal conductivity, whose behavior is governed by the following system of non-linear differential equations

$$\rho \frac{\partial v_i}{\partial t} + \rho_{,i} + \rho v_k v_{i,k} = 0, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \rho_{,k} v_k + \rho v_{k,k} = 0, \tag{2}$$

$$\frac{\partial \rho}{\partial t} - \rho v_k \frac{\partial v_k}{\partial t} - \rho v_i v_k v_{i,k} + \gamma p v_{k,k} = 0, \tag{3}$$

where t denotes the time, p the pressure, ρ the density, v_i the components of velocity of the gas and γ is the gas constant. The above equations are referred to a system of rectangular coordinates x and the comma indicates partial differentation with respect to these coordinates in the usual manner. Relations (1) are the equations of motion and the relation (2) is the equation of continuity. The entropy condition[2] is expressed by the equation (3). In the special case for which the motion is stationary this latter equation gives the well known condition that the entropy is constant along stream lines although it may vary from stream line to stream line.[3]

A moving surface $\Sigma(t)$ is said to be singular of order 1 relative to the pressure, density and velocity if these quantities are continuous across the surface but at least one of the first derivatives of p, ρ, or v_i with respect to the coordinates

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[2] See, T. Y. THOMAS, *The fundamental hydrodynamical equations and shock conditions for gases*, Math. Mag. 22 (1949) pp. 169–189.

[3] In the stationary case the relation between the pressure and density of the gas is given by $p = N\rho^{\gamma}$ where N is a function which is constant along stream lines. A specialization of this case arises when we assume that N is constant throughout the gas, or, more generally, if we assume an equation of state of the form $p = f(\rho)$ relating the pressure and the density. It may be noted in this connection that the method of this paper is applicable, with trivial modifications, when the equation (3) is replaced by the above equation of state provided the function $f(\rho)$ is continuous and twice differentiable.

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