

Spinor Geometry and General Field Theory

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Introduction. The development of geometrical (unified) field theories in the past forty years has been dominated by the idea of generalized metrics and vector connections; spinors have played a comparatively subordinate role. Although spin connection was frequently considered, particularly in connection with the formulation of DIRAC'S wave equation in general and projective relativity, it was usually treated as a secondary structure superimposed upon the primary metrical structure of the base manifold.¹

The purpose of the present work is to exploit, more fully than it has been done heretofore, the possibilities of spin connection from the point of view of geometrical field theories and to develop a geometry² whose connection is derived exclusively from the displacement of spinors. There are two circumstances in particular—one geometrical and one physical—which seem to encourage such a development. First, spinors are decidedly more “primitive” geometrical objects than Euclidean vectors, as seen for instance from the fact that, unlike Euclidean rotations, spin transformations represent the full covering group of the orthogonal group of degree $n \geq 3$. Second, spin connection is known to introduce essentially new elements into the metrical structure which can eventually be interpreted as the geometrical equivalents of the electromagnetic and possibly other physical fields.³

The spinor concept to be used is of considerable importance. The spin representation of the 4-dimensional pseudo-orthogonal (Lorentz) group \mathcal{L} is usually obtained from a (real) Clifford algebra Ω with four generators; the even elements

¹ For a bibliography on spinor analysis in general and projective relativity prior to 1953 the reader is referred to [1]. Of references not included there we mention HLAVATÝ [6] and INGRAHAM [7]. The former deals with spin connection in manifolds with non-symmetrical metric tensor, the latter with spin connection in conformal relativity. INGRAHAM'S treatment has a certain affinity to the present one as it is fundamentally based on spin connection.

² By a geometry we understand a structure impressed upon a differentiable manifold and consisting of (i) a fibre bundle with suitable differentiability properties and (ii) a connection established by the virtual displacement of the fibre over the manifold.

³ The fact that spin connection may introduce new elements into the geometrical structure was first noted by FOCK [5]. FOCK speaks of the “geometrizing” of DIRAC'S equation; the electromagnetic vector potential appears as part of the spin connection.