

Lie Algebras of Classical Type

W. H. MILLS & G. B. SELIGMAN

Communicated by N. JACOBSON

The purpose of this paper is to present a unified technique for the classification of those semi-simple Lie algebras to which the methods of KILLING and CARTAN are applicable. Five axioms for such a classification theory are presented, and from them the customary sets of invariants (matrices of integers or Schläfli diagrams) are developed and their completeness proved. The argument from the axioms to the isomorphism (completeness) theorem is elementary, involving only some familiarity with finite-dimensional vector spaces. The first two axioms require that the algebra have no center and that it be its own derived algebra. The other three require the existence of a Cartan decomposition with certain special properties. Among the algebras satisfying these axioms are all semi-simple Lie algebras over algebraically closed fields of characteristic zero (see [5]) and those algebras over algebraically closed fields of characteristic $p > 7$ that are treated in [8]. In particular, the axioms are satisfied by all Lie algebras with non-degenerate Killing forms over algebraically closed fields of characteristic different from 2, 3, 5.

The previous treatments of the classification theory all depend heavily on the existence of a non-degenerate bilinear form with certain properties, but unfortunately it is not clear whether or not a suitable form actually exists for all the Lie algebras obtained. Thus it is desirable to have a complete treatment that does not depend on such a form. Our present paper provides such a treatment.

Apart from giving the theory a basis which seems close to the most general setting in which it is applicable, it is to be hoped that the authors have achieved sufficient conciseness to make the results of [8] for non-zero characteristic more accessible. It will be noted that the multiplicity of case distinctions of that paper has been considerably reduced by the development of more general principles.

The existence of a full collection of algebras, one for each system of invariants, has long been known for characteristic zero ([1], [5], [7], [9], [11]). A complete answer to this question for $p \neq 0$ was not supplied in [8]. Using some of CHEVALLEY'S results on the choice of structure constants at characteristic