

# Classical Type Lie Algebras of Characteristic 5 and 7

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In a companion paper [C] to this one\*, SELIGMAN and I, starting from a suitable set of axioms, have developed a classification theory of Lie algebras—the algebras obtained are the semi-simple Lie algebras over algebraically closed fields of characteristic zero and their analogues over all other fields of characteristic not 2, 3, 5, 7.

In this paper it is shown that the results of [C] can be extended to fields of characteristic 5 and 7.

**1. Preliminaries.** Let  $\mathfrak{L}$  be a finite dimensional Lie algebra over a field  $\mathfrak{F}$  of characteristic  $p$ , and suppose that  $\mathfrak{L}$  contains an abelian Cartan subalgebra  $\mathfrak{H}$ . Let  $\mathfrak{H}^*$  denote the dual space of  $\mathfrak{H}$ . For any  $\alpha \in \mathfrak{H}^*$  let  $\mathfrak{L}_\alpha$  denote the set of all  $a \in \mathfrak{L}$  such that  $[ah] = \alpha(h)a$  for all  $h \in \mathfrak{H}$ . If  $\mathfrak{L}_\alpha \neq 0$ , then  $\alpha$  is called a root. We assume that  $\mathfrak{L}$  and  $\mathfrak{H}$  satisfy the five axioms of [C], namely:

- (i)  $[\mathfrak{L}\mathfrak{L}] = \mathfrak{L}$ .
- (ii)  $\mathfrak{L}$  has a trivial center.
- (iii)  $\mathfrak{L}$  is the direct sum of the subspaces  $\mathfrak{L}_\alpha$ .
- (iv)  $[\mathfrak{L}_\alpha\mathfrak{L}_{-\alpha}]$  is one-dimensional for every non-zero root  $\alpha$ .
- (v) For every non-zero root  $\alpha$  and every  $\beta \in \mathfrak{H}^*$  there is a positive integer  $m$  such that  $\beta + m\alpha$  is not a root.

We note first that there must be a non-zero root  $\alpha$  by (iii) and (i), that  $-\alpha$  is also a root by (iv), that 0 is a root, and hence  $p \neq 2, 3$  by (v).

The Properties (vi)—(xix) of [C] are all valid for  $p \neq 5$ ; the proofs given in [C] hold. For  $p = 5$  the proofs of these properties given in [C] hold except for those of Properties (xvii) and (xviii). In this case Property (xviii) is an immediate consequence of Axiom (v). On the other hand Property (xvii) is actually false for

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\* W. H. MILLS & G. B. SELIGMAN, Lie algebras of classical type, THIS JOURNAL pp. 519–548. The letter [C] will be used throughout to refer to this paper.