

# *Low Reynolds Number Flow Past a Circular Cylinder\**

SAUL KAPLUN

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The ideas discussed in Ref. 1 are applied to the problem of finding a uniformly valid asymptotic expansion of the Navier-Stokes solution for incompressible stationary flow past a circular cylinder at low Reynolds numbers. Some additional remarks are made about the general two-dimensional problem. The following notation is used:  $L$  is a characteristic length of the object, in particular the circular cylinder is defined by

$$r^2 = x_1^2 + x_2^2 = \left(\frac{L}{2}\right)^2. \quad (1)$$

At infinity the velocity  $\mathbf{q}$  is  $U\mathbf{i}$  and the pressure  $p$  is  $p_\infty$ . The kinematic viscosity is  $\nu$ . The following non-dimensional quantities are introduced

$$\text{Reynolds number} = \text{Re} = \frac{UL}{\nu}, \quad \mathbf{q}^* = \frac{\mathbf{q}}{U}, \quad (2a)$$

$$\tilde{x}_i = \frac{x_i U}{\nu}, \quad \tilde{r} = \frac{r U}{\nu} \quad (\text{Oseen variables}), \quad (2b)$$

$$x_i^* = \frac{x_i}{L} = \frac{\tilde{x}_i}{\text{Re}}, \quad r^* = \frac{r}{L} = \frac{\tilde{r}}{\text{Re}} \quad (\text{Stokes variables}), \quad (2c)$$

$$\nabla^* = \left( \frac{\partial}{\partial x_1^*}, \frac{\partial}{\partial x_2^*} \right), \quad \nabla \approx \left( \frac{\partial}{\partial \tilde{x}_1}, \frac{\partial}{\partial \tilde{x}_2} \right). \quad (2d)$$

**Result.** The velocity field has the following asymptotic representation uniformly valid for small  $\text{Re}$ :

$$\begin{aligned} \mathbf{q}^* &= \mathbf{S}_m + O(\epsilon^{m+1}), \\ \mathbf{S}_m &= \{\mathbf{i} + \epsilon[\mathbf{g}_1(\tilde{x}_i) + \mathbf{l}_1(x^*)]\} + \sum_{n=2}^m \epsilon^n \{\mathbf{k}_n(\tilde{x}_i) + a_n \epsilon[\mathbf{g}_1(\tilde{x}_i) + \mathbf{l}_1(x^*)]\} \end{aligned} \quad (3)$$

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