

Energy Integrals for the Mixed Problem in Hyperbolic Partial Differential Equations of Higher Order

GIDEON PEYSER

Communicated by W. MAGNUS

Introduction. In this paper we shall discuss the mixed initial and boundary value problem of the linear hyperbolic partial differential equation of higher order in two dimensions: x — space dimension, y — time dimension. We approach the problem through a basic energy integral inequality, developed in Section 2, which provides an *a priori* estimate of the solution of the given equation.

Any solution of a hyperbolic differential equation necessarily consists of three distinct parts: Uniqueness, existence and continuability of the initial conditions. Initial conditions are called continuable, see FRIEDRICHS [5], if a solution of the equation exists such that the type of conditions imposed on the initial data for $y = 0$ holds also for appropriate space-like curves, *e.g.* for $y = y_0 > 0$. In our case of the mixed initial and boundary data we consider also another part, the continuability of the boundary conditions. By continuability of the boundary conditions we mean that the type of conditions imposed on the boundary data for $x = 0$ holds also on appropriate time-like curves, *e.g.* $x = x_0 > 0$. L. CAMPBELL & A. ROBINSON [1] treated the uniqueness and existence of the mixed hyperbolic problem by reduction to systems of ordinary differential equations.

Our use of energy integrals requires work in certain L_2 spaces and results in generalized “strong” solutions of the given equations. THOMÉE [14] developed energy integral estimates for the special case of the mixed problem for the hyperbolic equation in two dimensions of third order with constant coefficients. Energy integrals were introduced by FRIEDRICHS & LEWY [6] (also ZAREMBA [15]). LERAY [9] and GÂRDING [7] arrive at energy estimates for the Cauchy problem of the hyperbolic equation of higher order in several variables by multiplying the given hyperbolic differential operator of order $m + 1$ by another operator

This paper is essentially a dissertation submitted to the faculty of New York University. The author wishes to express his gratitude to Professors P. D. LAX and L. NIRENBERG for their helpful suggestions and criticism.