

On Stability of Periodic Solutions of Weakly Nonlinear Differential Systems

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1. In previous papers R. A. GAMBILL & J. K. HALE [9] and L. CESARI & J. K. HALE [5] have given general existence theorems for periodic solutions (harmonics, subharmonics) of periodic weakly nonlinear differential systems. The method used in these papers is the convergent method of successive approximations which has been successively developed by L. CESARI, R. A. GAMBILL and J. K. HALE in a series of papers. In particular, in [12] general existence theorems have been given for periodic solutions (cycles) of autonomous weakly nonlinear differential systems; in [3, 4, 7, 8, 11, 13] convergent series expansions have been studied for the characteristic exponents of periodic linear differential systems, and, consequently, criteria have been given for the boundedness or unboundedness of their solutions.

In the present paper the same method, together with a theorem of LIAPOUNOFF, is applied to derive a criterion for asymptotic stability of periodic solutions of periodic weakly nonlinear differential systems as obtained in [5] and [9]. The systems discussed are introduced in §3. The final result is given in §5 and extends, to such systems, a criterion given by L. MANDELSTAM & N. PAPALEXI [15, 16] for a second order differential equation. The criterion is formally similar to the one given by a different approach in [16, p. 153 ff.] for autonomous systems.

The consistent use of the method has advantages. On the one hand the criterion for asymptotic stability of a periodic solution is cast into expressions some of which already appear in the approximation of the periodic solution; on the other hand the two aspects of the method mentioned (determination of periodic solutions of the given nonlinear system and determination of characteristic exponents of the ensuing periodic linear variational system) are taken into consideration.

Examples are discussed in §6: The well known van der Pol equation with forcing term and the rather difficult coupling of two such van der Pol equations. Other examples will be discussed in other papers.

For references on this vast subject we refer to the papers listed in the bibliography.