

# *Imbedding of Compact, Differentiable Transformation Groups in Orthogonal Representations*

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**1. Introduction.** Let  $G$  be a Lie group and  $M$  a differentiable (*i.e.*  $C^\infty$ ) manifold. An *action* of  $G$  on  $M$  is a homomorphism  $\varphi : g \rightarrow \varphi_g$  of  $G$  into the group of diffeomorphisms (*i.e.* non-singular, differentiable homeomorphisms) of  $M$  with itself such that the map  $\Phi : (g, x) \rightarrow \varphi_g(x)$  of  $G \times M$  into  $M$  is continuous (it then follows from a theorem of BOCHNER & MONTGOMERY [1] that  $\Phi$  is automatically differentiable). A triple  $(G, M, \varphi)$  of such objects as those above we shall call a differentiable  $G$ -transformation group. If  $O$  is an open submanifold of  $M$  invariant under each  $\varphi_g$  then  $g \rightarrow \varphi_g|_O$  is an action of  $G$  on  $O$  and we will denote by  $(G, O, \varphi)$  the corresponding differentiable  $G$ -transformation group. If  $(G, M, \varphi)$  and  $(G, N, \psi)$  are two differentiable  $G$ -transformation groups then an *imbedding* of  $(G, M, \varphi)$  in  $(G, N, \psi)$  is an imbedding  $f$  of  $M$  into  $N$  such that  $f \circ \varphi_g = \psi_g \circ f$  for all  $g \in G$ .

A particular class of differentiable  $G$ -transformation groups consists of the finite dimensional orthogonal representations of  $G$ , *i.e.* triples  $(G, M, \varphi)$  where  $V$  is a finite dimensional real Hilbert space and each  $\varphi_g$  is an orthogonal transformation of  $V$ . We shall prove the following theorem, and in fact something slightly more general.

**Theorem.** *A differentiable  $G$ -transformation group  $(G, M, \varphi)$  can be imbedded in a finite dimensional orthogonal representation of  $G$  provided  $G$  and  $M$  are compact.*

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\*After this work was completed, the author was informed of a more general theorem of G. D. Mostrow, in which differentiability and compactness of  $M$  are not assumed. While Mostrow's theorem is deeper and more difficult to prove, the simplicity gained by assuming differentiability seems to justify the publication of this paper. (Added in proof: Mostrow's paper, which appeared under the title "Equivariant imbeddings in Euclidean space" in the May, 1957 issue of the *Annals of Mathematics*, contains a special proof for the differentiable case which, though very different from ours, is quite short.)