

The Decay of Waves in Elastic Solids

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1. Introduction. Consider an isotropic elastic solid with displacements from the unstrained position given by $u_i(x, t)$ where the x 's are the coordinates of a dynamically allowable rectangular system and t denotes the time. The equations of motion are

$$(1) \quad \sigma_{ii,i} = \rho \frac{dv_i}{dt},$$

where the quantities σ_{ij} are the components of the symmetric stress tensor such that

$$(2) \quad \sigma_{ij} = \lambda u_{k,k} \delta_{ij} + \mu(u_{i,j} + u_{j,i}),$$

in which λ and μ are the well known material constants of the medium. Also in (1) the quantity ρ is the density and the v_i are the components of velocity. As customary we assume for simplicity that the v_i are given by the formula

$$(3) \quad v_i = \frac{\partial u_i}{\partial t},$$

which can be applied under the condition that the displacements u and their spatial derivatives are sufficiently small. Indeed, this condition is one of the essential requirements of the linear or classical theory of elasticity with which we are concerned in this article.

It has been known for over a hundred years that two types of waves having different velocities can be propagated within an isotropic elastic medium¹. The names *irrotational* and *equivoluminal* have been given to these waves for reasons which will be apparent from the discussion in the following section. It is the purpose of the present article² to investigate the question of the decay in the strength of the irrotational and equivoluminal waves during propagation on

¹ For reference to the early literature see A. E. H. LOVE, *A Treatise on the Mathematical Theory of Elasticity*, 4th. Ed., New York, Dover Publications, 1944.

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