

Some Purely Deterministic Processes

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1. Introduction. The linear prediction problem as it arises in the case of stationary processes has attracted much attention. The problem has been studied most intensively when the prediction error is known to be positive. It is of some interest to consider a few simple situations in which the prediction error is zero, especially as a situation of this sort arises in NEUMANN's theoretical model of storm-generated ocean waves [3]. I shall, unfortunately, not be able to discuss the problem that arises in the context of NEUMANN's model in any detail.

Let x_t , $Ex_t \equiv 0$, be a weakly stationary process, that is,

$$r_{t,\tau} = Ex_t x_\tau = r_{t-\tau}$$

depends only on the time difference $t - \tau$. Our process is assumed to be real-valued. The time parameter t may either range over the real numbers or it may range over the integers. The first case is the continuous parameter case and the second is that of a discrete parameter. Examples of both continuous parameter and discrete parameter processes will be discussed. The discrete parameter processes discussed will be of much greater interest than the continuous parameter processes.

2. Preliminary Remarks. If x_t , $Ex_t \equiv 0$, is a weakly stationary process, it has a random Fourier representation of the form

$$(1) \quad x_t = \int_{-\infty}^{\infty} e^{it\lambda} dZ(\lambda), \quad E dZ(\lambda) \overline{dZ(\mu)} = \delta_{\lambda\mu} dF(\lambda)$$

in the continuous parameter case $-\infty < t < \infty$, and a random Fourier representation

$$(2) \quad x_t = \int_{-\pi}^{\pi} e^{it\lambda} dZ(\lambda), \quad E dZ(\lambda) \overline{dZ(\mu)} = \delta_{\lambda\mu} dF(\lambda)$$

in the discrete parameter case $t = \dots, -1, 0, 1, \dots$. Here $\delta_{\lambda\mu}$ is the Kronecker delta

$$\delta_{\lambda\mu} = \begin{cases} 1 & \text{if } \lambda = \mu \\ 0 & \text{if } \lambda \neq \mu. \end{cases}$$