

Uniqueness and Representation Theorems for Solutions of $\Delta u + k^2 u = 0$ in Infinite Domains

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1. Introduction. In this paper we will discuss uniqueness and representation theorems for solutions u of the reduced wave equation

$$(1) \quad \Delta u + k^2 u = 0, \quad k > 0,$$

when the domain D of definition of u is infinite. B denotes the boundary of D .

HISTORY OF THE PROBLEM.

We will discuss first the case in which B is bounded. In 1912 SOMMERFELD [8] introduced the radiation and boundedness conditions for u . These are respectively

$$(2) \quad \lim_{r \rightarrow \infty} \frac{1}{2} r (n - 1) \left| \frac{\partial u}{\partial r} - iku \right| = 0,$$

$$(3) \quad \lim_{r \rightarrow \infty} \frac{1}{2} r (n - 1) |u| < \text{constant.}$$

n is the dimension and r is a radial polar coordinate measured from some fixed point in space. Using (2) and (3) and a special assumption concerning the Green's function for (1), SOMMERFELD proved the uniqueness of the exterior boundary value problem for u . In 1942 W. MAGNUS [2] deduced the same result using only (2) and (3). In 1943 RELICH [5] obtained the uniqueness theorem using only the radiation condition and, moreover, in the following weakened form:

$$(4) \quad \lim_{R \rightarrow \infty} \iint_{\Sigma_R} \left| \frac{\partial u}{\partial R} - iku \right|^2 dS = 0$$

where Σ_R is the surface of a sphere of radius R with some fixed point in space as center and dS is a surface element. He obtained his proof by showing that

$$(5) \quad \lim_{R \rightarrow \infty} \iint_{\Sigma_R} |u|^2 dS \neq 0$$