

# *Symmetric Bending of Shallow Shells of Revolution\**

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*Communicated by* T. Y. THOMAS

**1. Introduction.** The present paper is concerned with the problem of symmetrical bending of thin elastic shells of revolution for shells which at the same time may be considered as shallow shells. We assume a somewhat more general system of stress strain relations than is usually done in shell theory and for this system reduce the problem to two simultaneous second-order non-linear differential equations. For an isotropic shell, which is homogeneous in wall thickness direction, and without temperature strains, these equations become special cases of general shallow shell theory [1] and also of earlier results for non-shallow shells [2].

Of the many possible applications of the general equations obtained in this paper we consider first the problem of the edge effect in shallow spherical shells subjected to internal pressure. For this effect there exist two well known solutions, one within the framework of linear bending theory [3] and the other within the framework of non-linear membrane theory [4]. The connection between these two theories, and, in fact, a more general theory in which linear bending theory and non-linear membrane theory appear as limiting cases, has previously been considered by the writer [2]. The present treatment reproduces our earlier results for the edge effect in somewhat greater detail and supplements them by numerical calculations.

As a second problem we consider the effect of uniform surface pressure in accordance with linear theory, assuming different elastic behavior in meridional and circumferential direction. We find that the nature of the solution of the problem is altered in a fundamental way by going from an isotropic material to an orthotropic material. While linear membrane theory furnishes an appropriate solution for the interior of the isotropic shell, this solution becomes singular in an inadmissible way for the interior of the orthotropic shell. Accord-

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\*A report on work supported by the Office of Naval Research of the United States Navy under a Contract with the Massachusetts Institute of Technology.