

# *On the Principle of Invariant Imbedding and Neutron Transport Theory I—One-Dimensional Case*

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**1. Introduction.** In a series of notes, [1], [2], [3], we have sketched some applications of the Principle of Invariant Imbedding, and the associated functional equation approach, to an important class of problems in radiative transfer, a field pioneered and extensively cultivated by AMBARZUMIAN and CHANDRASEKHAR, [4], and to a class of problems in neutron diffusion theory.

In this paper we wish to develop in some detail the techniques presented in the third note above, [3]. We shall consider a one-dimensional version of the reflection and transmission of neutrons, and the phenomenon of critical mass. Although we shall consider the effects of fission, capture and forward and backward scattering, we shall not take into account collisions between neutrons.

Our basic model will be stochastic, with standard transport theory derived on the basis of expected values. As the explicit form of the higher moments indicates, an expected value theory yields a quite inadequate view of the distribution of reflected and transmitted neutrons in the neighborhood of critical mass.

We shall present some numerical results describing these distributions obtained from the solution of forty simultaneous nonlinear differential equations. Problems of this nature, once so formidable, can be done in a routine fashion with modern digital computers.

In particular, we would like to emphasize the fact that the determination of critical length in the one-dimensional model, and critical dimensions in the general case, is made to depend upon the solution of nonlinear differential equations with one-point conditions, initial conditions, rather than upon the solution of two-point boundary value problems which involve characteristic values.

In subsequent publications, we will apply the same techniques to multi-dimensional models, and discuss in detail the connection between the various modes of approach.

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