

Potential Theory and a Generalized Jensen-Neumanlinna Formula for Functions of Several Complex Variables

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1. Introduction. One of the striking differences between bounded domains D in complex n -space for $n = 1$ and for $n > 1$ is in the nature of the smallest closed subset B (*the Bergman-Šilov boundary*) of the closure \bar{D} of D on which all complex functions analytic on \bar{D} assume their maximum modulus. B is the whole topological boundary if $n = 1$, while if $n > 1$, B may be a proper subset of the topological boundary. Another difference lies in the fact that for $n > 1$ the real parts of analytic functions are not a dense subset of the space of continuous real functions on B . Because of these two differences, the methods of the theory of functions of several variables have rarely generalized the potential theoretic methods which have become classical in the theory of functions of one complex variable.

S. BERGMAN [1] has formulated the problem of finding classes \mathcal{E} of functions including the real parts of analytic functions, the functions of each having their maxima on B , and, if possible, such that each class has boundary values dense in the space of continuous functions on B . In a previous paper [10] I examined this problem for a special class of bounded domains found by E. CARTAN: the symmetric homogeneous ones. Such a domain (*a Cartan domain*) has a transitive group of complex analytic transformations (*automorphisms*) including, for each point p , an automorphism with p alone as fixed point. It was shown that the additional requirement that the class \mathcal{E} be carried into itself by automorphisms makes \mathcal{E} unique. This class of functions is given by an integral formula of simple structure generalizing the Poisson integral. What is most important, the class \mathcal{E} is the class of solutions of a certain second order elliptic partial differential equation $\Delta f = 0$, where Δ is the Laplace-Beltrami

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