

# *On Singular Integral Equations with Logarithmic or Cauchy Kernels*

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**1. Introduction.** Many of the boundary problems for elliptic partial differential equations in two variables can be reduced to one-dimensional integral equations with logarithmic or Cauchy kernels. Unless the geometry be such that the latter are over infinite or semi-infinite ranges an explicit solution is in general not possible and one is led to the question of numerical approximations.

The standard device for solving integral equations numerically is to replace all functions occurring by polynomial approximations. If the kernels are analytic this is satisfactory and even for singular kernels is still feasible [1]. Polynomial approximations conceal, however, the singular nature of the solutions at the ends of the interval of integration. Since it is precisely this singular behavior which is often of interest in the original boundary problem we shall investigate it here for certain classes of integral equations. We leave aside entirely questions of existence of solutions of these equations since such questions are best discussed within the framework of the differential equation problem they represent.\*

Our general method is the following. We shall translate the integral equations into functional relations for multi-valued analytic functions of a complex variable. Without being able to solve these relations we can still obtain information about the structure of the analytic function, which information we translate back into facts concerning the solutions of the integral equations. The idea of introducing complex variables dates back to CARLEMAN [3] and [4] who was able, in this manner, to obtain explicit solutions of some very elementary equations. The necessary tools for a systematic investigation of a large class of integral equations were presented by LEWY [5] and [6] in his study of difference-differential equations.

We study equations of the following three types, all for  $0 < x < 2$ :†

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\* A discussion of such singular integral equations is given in [2].

† All integrals which appear should be interpreted in the Cauchy principal value sense. We make the upper limit 2 in all our integrals as a numerical convenience for Section 7. This can of course always be achieved by a change in scale. With analytic  $A(x,t)$  it is clear that we could replace  $A(x,t)$  by  $A(x,x)$  and merely change  $C(x,t)$ .