

*On Elliptic Partial Differential Equations and Uniqueness Theorems for Closed Surfaces*¹

PHILIP HARTMAN

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1. Theorems in differential geometry. This paper deals with the conditions of differentiability for the validity of several theorems concerning closed, 2-dimensional surfaces in Euclidean 3-space. The emphasis in this paper is on the portion of the proofs involving elliptic partial differential equations. In particular, most of the paper deals with the behavior of a solution of a linear, elliptic partial differential equation in the vicinity of a zero.

Consider the following uniqueness statement:

(A) Let $F(H, K, \lambda, \mu, \nu)$ be defined on the set $H > 0, H^2 \geq K > 0, \lambda^2 + \mu^2 + \nu^2 = 1$ and satisfy

$$(1) \quad \frac{\partial F}{\partial k_1} \frac{\partial F}{\partial k_2} > 0, \quad \text{where} \quad 2H = k_1 + k_2 \quad \text{and} \quad K = k_1 k_2.$$

Then, up to translations, there is at most one closed surface S of positive Gaussian curvature such that the mean and Gaussian curvatures, H and K , at the point of S where the inward unit normal of S is (λ, μ, ν) satisfy the relation $F = 0$.

This assertion is due to A. D. ALEXANDROFF when F and S are analytic. It was proved by POGORELOFF under the conditions that F and S are sufficiently differentiable. His assumptions have been lightened successively by WINTNER and myself ([9], where earlier references are given) and by BERS [2]. In both [2] and [9], the assumptions imply that S is of class C^3 (in fact, S has local parametrizations in terms of functions possessing Hölder continuous third derivatives). The question was raised in [9] whether (A) holds under hypotheses which do not imply that S is of class C^3 . This question will be answered below in the affirmative.²

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²Added March 17, 1958. Since this paper has been submitted, additional results of ALEXANDROFF [*Vestnik*, Leningrad University, 1956, No. 19, pp. 1-17] and POGORELOFF