

Interior Estimates for Parabolic Systems of Partial Differential Equations

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Introduction. Interior estimates for second order elliptic equations have been derived by J. SCHAUDER [8][9]. More recently A. DOUGLIS & L. NIRENBERG [2] have extended these estimates to elliptic systems of great generality. Their method, specialized to second order equations, is a simplification and modification of SCHAUDER's method, some of the ideas occurring already in an earlier paper of E. HOPF [6]. They have also used their interior estimates to prove differentiability of solutions of elliptic systems, both linear and nonlinear.

Interior estimates for solutions of second order parabolic equations were derived by RICHARD B. BARRAR in his unpublished thesis in 1952. His method is patterned on SCHAUDER's, the results also being completely analogous to SCHAUDER's results. The special case of one space-dimension was derived more recently also by C. CILIBERTO [1].

In this paper we shall obtain interior estimates analogous to SCHAUDER's for solutions of general parabolic systems. Our method is patterned on that of DOUGLIS & NIRENBERG rather than on SCHAUDER's and, therefore, specialized to second order parabolic equations, it presents a simplification of the results of BARRAR and CILIBERTO.

In §1 we introduce an appropriate distance-function in the (x, t) -space and define accordingly the notion of Hölder continuity. Then we introduce norms in (1.5), (1.6), the latter being somewhat different from the analogous norm in [2]. We mention several elementary but useful lemmas.

In §2 we define the notion of linear parabolic systems and obtain estimates for the solutions of homogeneous systems with constant coefficients (Theorem 1). Theorem 1 is the analogue of a potential-theoretic theorem [2; Theorem 2]. Most ideas in the proof occur in [2] and [6], but some technical differences arise from the different nature of the singularity of the fundamental solutions.

In §3 we prove the main result of this paper (Theorem 3). The proof is analogous to that of [2], via Theorem 1 of §2.

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