

# *On Rigidity Properties Of Developable Surfaces*

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## INTRODUCTION

Consider a surface  $S$  containing a curve  $C$ . Let us imagine that  $S$  is made of some flexible but inextensible material. Then  $S$  is capable only of isometric deformations (transformations under which arc-length on  $S$  is invariant). If one further assumes that the curve  $C$  on  $S$  is kept point-wise fixed, then the possible deformations of  $S$  are further restricted, and for certain choices of  $S$  and  $C$  it may happen that all these restrictions imply that  $S$  is rigid (is capable of no deformations at all). Questions of this type are related to issues in the theory of rigid surfaces (for a general account of this theory, see, for example, [1]). However, in the theory of rigid surfaces most of the results are concerned with infinitesimal rigidity (see, for example, [2], [3]) and there arises the question whether analogous results are valid for rigidity concepts which are more closely related to physically relevant approaches. The purpose of this note is to study this question for the case of developable surfaces  $S$ .

In this type of study, the regularity assumptions concerning the curves, surfaces, and deformations under consideration are essential, as simple examples show. The regularity assumptions relate partly to differentiability assumptions and partly to the absence of geometrical singularities. We shall have to be quite careful in the following discussion as regards the exclusion of geometrical singularities, and, in any case, more careful than is customary in differential geometry. On the other hand, to keep the presentation reasonably brief, we shall assume that all the functions and transformations occurring in the sequel are continuously differentiable to the extent sufficient for our purposes. The determination of the precise necessary and sufficient differentiability conditions in this sort of study may lead to quite interesting issues (see, for example, [2], [3]), and we hope to elaborate this point on another occasion.

## 1. PRELIMINARIES

**1.1. Notations.** Vector notation will be used throughout. If  $a, b$  are vectors, then  $ab$  and  $a \times b$  are the scalar product and the vector product of  $a$  and  $b$