

*Some Limit Theorems for Stochastic Processes*¹

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1. Introduction. The purpose of this paper is to study the asymptotic behavior, for large values of t , of several random variables associated with certain types of stochastic processes. The simplest type of process considered is a "recurrent event" in discrete time. (See [7] or [8] for the definition and terminology of recurrent events; other terms will be defined precisely in Section 2 of this paper.) The random variables to be studied are Y_n , the time elapsed (measured at time n) since the last occurrence of the event, Z_n , which is the time until the next occurrence, and T_n , the time between occurrences. It turns out (Theorem 3.2) that the limiting distribution of Y_n/n must be a "generalized arc sine" law; the condition for the existence of this limit distribution is also a familiar one.²

A recent paper of DYNKIN [6] contains results very closely related to those of this paper. He studies sums of positive, independent, identically distributed random variables X_m , and finds the condition for $\xi(t)/t$ to have a limiting distribution where

$$(1.1) \quad \xi(t) = \min \{S_m \mid S_m = X_1 + \cdots + X_m \geq t\} - t.$$

Suppose that X_m is the time between the $(m - 1)^{\text{th}}$ and the m^{th} occurrence of a certain recurrent event E , so that $S_m = X_1 + \cdots + X_m$ is the time of the m^{th} occurrence. These random variables X_m are positive, independent and identically distributed, and $\xi(n)$ defined by (1.1) with these X 's can be identified with Z_n for the event E . Since (as DYNKIN points out; also see Section 5 of this paper) it is easy to pass from the distribution of Z_n/n to that of Y_n/n and *vice versa*,

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² The generalized arc sine laws are the limiting distributions of the relative number of positive partial sums in the addition of independent, identically distributed random variables; this problem has been studied by ANDERSEN and, more completely, by SPITZER [12]. The condition referred to arises in studying the time during which a recurrent event occurs (FELLER, [7]).