The Method of Characteristics for a Perfect Compressible Fluid in General Relativity and Non-steady Newtonian Mechanics*

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1. Introduction. In the case of the non-steady flow of a compressible fluid in Newtonian mechanics, the known expression for the characteristic partial differential equation (the first of the characteristic relations) does not lead to any simple geometric relation between the three-dimensional velocity vector and the vectors of the normal and bicharacteristic cones. This is so whether we consider these cones as lying in a three-dimensional Euclidean space or as lying in a four-dimensional Euclidean space-time manifold [1]. It should be noted that in the steady case the velocity vector lies along the axis of these cones which are right circular cones [2]. The difficulty in the non-steady case is that the velocity vector lies in a three-dimensional space but the generators of the above cones lie in a four-dimensional space. Another way of stating this result is that Newtonian mechanics does not possess space-time invariance. Thus it appears desirable to determine the first characteristic relation and then the complete characteristic system in the theory of general relativity.

In addition to the value of the characteristic system of general relativity in studying the motions of compressible fluids in astrophysical problems, such a characteristic system can also be used to study motions of compressible fluids in Newtonian mechanics. This can be done in two ways by use of the usual approximation method for going from relativity theory to Newtonian mechanics. First, we can determine the characteristic system for the Newtonian case. Second, the reduction of solutions of the characteristic system in the relativistic case to the solutions in the Newtonian case can be studied. We shall develop the first method in this paper.

In the present paper we shall proceed as follows in order to obtain the character-

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