

# *On a Non-Linear Integral Equation\**

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1. In this paper a non-linear integral equation of the form

$$y(t) = \int_0^t \frac{G[y(\tau), \tau]}{\sqrt{\pi(t-\tau)}} d\tau$$

is studied. Such an equation arises in connection with the problem of determining the temperature of a semi-infinite rod whose end is heated by a source of (known) variable temperature  $\phi(t)$ . The rod is assumed to be initially at temperature 0 throughout. The particular case of this problem where  $\phi(t) \equiv 1$  has been studied by W. R. MANN & F. WOLF [3] and by J. H. ROBERTS & W. R. MANN [4]. The temperature  $U(x, t)$  of the point  $x$  of the rod at time  $t$  satisfies the equations

$$(1) \quad U_{xx}(x, t) = U_t(x, t), \quad x > 0, \quad t > 0,$$

$$(2) \quad U(x, 0) = 0,$$

$$(3) \quad -U_x(0, t) = g[U(0, t), \phi(t)]$$

where  $g(u, v)$  denotes the rate of flow of heat from the source at temperature  $v$  to the end of the rod at temperature  $u$ .

In the special case where  $\phi(t)$  is bounded in  $t \geq 0$ , a probable hypothesis is the following:

$$(4) \quad |U(x, t)| \leq \text{some constant } M \text{ for } x \geq 0, t \geq 0.$$

As in [3] and [4], the rate of heat diffusion from the source to the end of the rod is assumed to follow a general non-linear law, so that the function  $g(u, v)$  satisfies the following conditions, which are clearly direct generalizations of the conditions on  $g$  assumed in [3] and [4]:

$$(5) \quad g(u, v) \text{ is continuous in both the variables } u \text{ and } v.$$

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