

*Integral Equations of the Schwarzschild-Milne Type**

PHILIP M. ANSELONE

Communicated by S. CHANDRASEKHAR

1. Introduction. The Schwarzschild-Milne integral equation of radiative transfer theory is

$$(1) \quad f(x) = \frac{1}{2} \int_0^\infty E_1(|x - y|) f(y) dy, \quad x \geq 0,$$

where

$$(2) \quad E_1(t) = \int_0^1 e^{-t/\mu} \mu^{-1} d\mu, \quad t > 0.$$

Auxiliary conditions are $f(x) \geq 0$ and $f(x) \equiv 0$. The integral equation was generalized by HOPF [1] and is further generalized in this paper.

Let K_1 be any function such that

$$(3) \quad K_1(t) \geq 0,$$

$$(4) \quad \int_{-\infty}^\infty K_1(t) dt = 1,$$

$$(5) \quad \int_{-\infty}^\infty K_1(t) t dt = 0,$$

$$(6) \quad \int_t^\infty K_1(\pm s) ds \leq CK_1(\pm t), \quad t \geq 0,$$

where C is some positive constant. These conditions are satisfied with $K_1(t) = \frac{1}{2}E_1(|t|)$ and $C = 1$. Let Λ denote the integral operator such that

$$(7) \quad (\Lambda f)(x) = \int_0^\infty K_1(x - y) f(y) dy, \quad x \geq 0,$$

whenever the right member exists for all $x \geq 0$. The problem defined by

$$(8) \quad f = \Lambda f, \quad f \geq 0, \quad f \equiv 0$$

is the announced generalization.

* Most of the material in this paper is taken from the author's 1957 Oregon State College doctoral dissertation.