

# *Kinematics of Homogeneous Turbulence*

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## INTRODUCTION

The idea of a statistical theory of turbulence goes back as far as J. BOUSSINESQ (1872) and O. REYNOLDS (1885); many of the ideas, now almost universally accepted, were introduced by Sir GOEFFREY TAYLOR between 1920 and 1935.† But it was really the creation, by the work of N. WIENER, P. LEVY and A. KOLMOGOROFF, of the modern theory of random functions which made possible a rigorous mathematical approach to the problem; the components of the velocity in a turbulent flow have to be considered as *random functions* of  $\mathbf{x}$  and  $t$ .

In this paper, we consider homogeneous turbulent velocity fields at a particular time  $t$ , limiting ourselves to the kinematics of the problem. Hence we are concerned with random vector fields (R.V.F.) defined in infinite 3-dimensional Euclidean space  $R^3$ —though for the sake of generality we consider R.V.F. with  $q$  components in  $p$ -dimensional space  $R^p$ .

There are several ways to define R.V.F. Since 1939 (see [8]), one of us has systematically considered an R.V.F. to be given by a vector function  $\mathbf{u}(\mathbf{x}, \omega)$  defined and measurable on the product space  $R^n \times \Omega$ , where  $\Omega$  is an unspecified *abstract* probability space. By choosing  $\omega$  in  $\Omega$ , one gets a *sample* velocity field. In [7], a systematic exposition of the kinematics of turbulence was given from this point of view, which has various mathematical advantages. It has, however, the disadvantage that the construction of such a function  $\mathbf{u}(\mathbf{x}, \omega)$  from observational data is not described.

Therefore, in this paper, a different procedure is followed. We consider first a *sample probability space*  $\Lambda$  (see [11, p. 155]), which consists of all velocity fields  $\mathbf{u}(\mathbf{x})$  which are measurable and have finite energy on compact sets. This class of fields seems sufficiently general. We define  $\Lambda$  as a topological linear space, and assume that the measure is “regular”, in the sense of being determined by the measures of Borel sets in  $\Lambda$ . In principle, one could consider the frequency

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†For a brief historical sketch, see [7, pp. 568–571].