

Degenerate Waves in Unsteady Gas Flow

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1. Introduction. In a previous paper [3], it was shown how to construct exact solutions of the quasi-linear partial differential equation for the velocity potential $\phi(x, y, t)$ in the unsteady plane flow of a gas by concentrating on degenerate flows of the type $\mathbf{q}(\lambda, t)$, $\lambda(x, y, t)$ being an unspecified variable. The method proposed for the integration of the equations of flow rested on the application of a contact transformation, with respect to the variables (x, y) , to the potential ϕ . The main results of this theory were that, in the physical plane, the curves of constant speed reduced to a system of straight lines; in general these lines were generated as tangents to an envelope which was not permitted to degenerate into a point. Therefore, for flows of the type considered, there existed no "simple centered wave" in the unsteady plane flow of a gas.

The purpose of the present paper is to develop an analogous theory of waves of the type $\mathbf{q}(\lambda, \mu)$, $\lambda(x, y, z, t)$, $\mu(x, y, z, t)$ in the unsteady spatial flow of a gas. In general, the acoustic speed does not reduce to a function only of the variables λ and μ ; in fact, it is shown in §2 that in general $a = a(\lambda, \mu, t)$. If, in the representation (λ, μ, t) , the acoustic speed becomes independent of t , then it is demonstrated in §10 that flows result from which may be derived "progressive plane waves" for which the velocity depends only on the quantities x/t , y/t , and z/t . This is briefly taken up in §§9, 10. GIESE [2] develops a similar theory for which $\mathbf{q} = \mathbf{q}(\lambda, \mu)$, $a = a(\lambda, \mu)$ without assuming isentropic conditions, referring to such flows as "double waves." Flows for which \mathbf{q} and a reduce to functions of three variables (λ, μ, ν) are referred to as "triple waves." Flows of the type $\mathbf{q}(\lambda, \mu)$, $a(\lambda, \mu, t)$ which are the subject of the present analysis therefore occupy an intermediate position between the double and triple waves of GIESE's theory. However, the title double waves is used for the flows here analyzed. A number of results connected with the properties of the hodograph and characteristic surfaces may be obtained for double waves, in GIESE's terminology; for details of these investigations we refer to GIESE's papers [1], [2].

It is well known that the Legendre transformation

$$\Phi = \phi - \mathbf{r} \cdot \mathbf{q}$$