

# *Invariant Imbedding and Neutron Transport Theory II—Functional Equations*

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## SUMMARY

Some one-dimensional fission processes are considered with particular regard to questions of critical length and energy dependence of the products of fission. Various interconnections between the functions used in the invariant imbedding approach and the classical approach are indicated. In particular, it is shown how one may pass from the consideration of certain linear two-point boundary value problems to nonlinear (Riccatian type) initial value problems. The former are useful for theoretical considerations, the latter for numerical computation.

## I. INTRODUCTION

In our two previous papers on the application of the principle of invariant imbedding to neutron transport processes, [3], [4], we considered the problem of determining probability distributions for the reflected and transmitted fluxes associated with one-dimensional rods. Using only expected values, these techniques led to methods for determining critical length.

The customary method of treating these questions, based upon the equation of transfer, leads to linear differential equations for the expected values, with two-point boundary conditions. The critical length is calculated as the solution of an eigenvalue problem.

The method presented in [3] and [4] is of an entirely different nature. The basic equations are now nonlinear differential equations whose solutions are determined solely by initial values. The critical length turns out to be the first value for which the solutions of the equations for expected flux become infinite.

In view of the fact that these two approaches to problems of this type appear to be so disparate, it is of some interest to link them, and to understand the

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